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INTEGRAL EQUATION OF THE THEORY OF LIFTING SURFACES, (U)  
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INTEGRAL EQUATION OF THE THEORY OF LIFTING SURFACES

By

N. N. Polyakhov



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**EDITED TRANSLATION**

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b><i>А а</i></b>	A, a	Р р	<b><i>Р р</i></b>	R, r
Б б	<b><i>Б б</i></b>	B, b	С с	<b><i>С с</i></b>	S, s
В в	<b><i>В в</i></b>	V, v	Т т	<b><i>Т т</i></b>	T, t
Г г	<b><i>Г г</i></b>	G, g	У у	<b><i>У у</i></b>	U, u
Д д	<b><i>Д д</i></b>	D, d	Ф ф	<b><i>Ф ф</i></b>	F, f
Е е	<b><i>Е е</i></b>	Ye, ye; E, e*	Х х	<b><i>Х х</i></b>	Kh, kh
Ж ж	<b><i>Ж ж</i></b>	Zh, zh	Ц ц	<b><i>Ц ц</i></b>	Ts, ts
З з	<b><i>З з</i></b>	Z, z	Ч ч	<b><i>Ч ч</i></b>	Ch, ch
И и	<b><i>И и</i></b>	I, i	Ш ш	<b><i>Ш ш</i></b>	Sh, sh
Й й	<b><i>Й й</i></b>	Y, y	Щ щ	<b><i>Щ щ</i></b>	Shch, shch
К к	<b><i>К к</i></b>	K, k	Ъ ъ	<b><i>Ъ ъ</i></b>	"
Л л	<b><i>Л л</i></b>	L, l	Ы ы	<b><i>Ы ы</i></b>	Y, y
М м	<b><i>М м</i></b>	M, m	Ь ь	<b><i>Ь ь</i></b>	'
Н н	<b><i>Н н</i></b>	N, n	Э э	<b><i>Э э</i></b>	E, e
О о	<b><i>О о</i></b>	O, o	Ю ю	<b><i>Ю ю</i></b>	Yu, yu
П п	<b><i>П п</i></b>	P, p	Я я	<b><i>Я я</i></b>	Ya, ya

\*ye initially, after vowels, and after Ъ, ь; e elsewhere.  
When written as ë in Russian, transliterate as yë or ë.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian      English

rot      curl  
lg      log

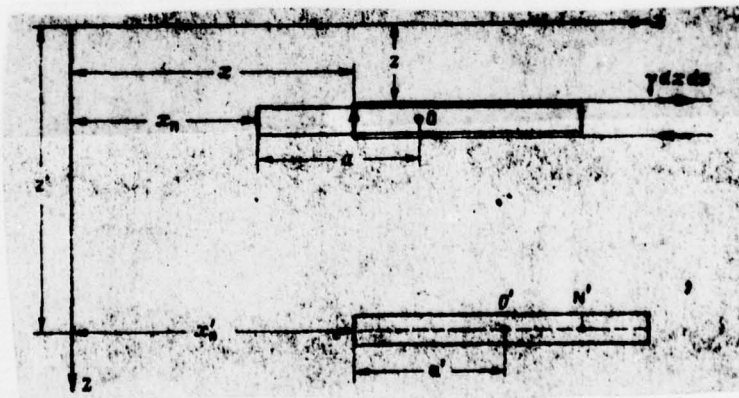


# INTEGRAL EQUATION OF THE THEORY OF LIFTING SURFACES

N. N. Polyakhov

This work, which is a continuation and a generalization of work [1], sets up a new form of an integral equation of the lifting surface and examines the method of its approximate solution.

§ 1. Setting up the basic equation. Let us assume that we have a supporting surface which represents a set of rectangular lines lying in the plane  $y=0$  and which are covered by a continuously distributed system of the  $\Pi$ -shaped vortices with density  $\gamma$ . We assume that the length of each line along the  $x$  axis is equal to  $2a$ , the width is equal to  $dz$ , and the span along the  $z$  axis is  $2L$ .



The inductive velocity caused by one  $\Pi$ -shaped vortex, located as shown in the figure, at point  $N'(x', 0, z')$  which lies in the middle of the line with number  $k$  is equal to [1]

$$\frac{\gamma(z, s) dz ds}{4\pi} \left\{ \frac{x' - x}{R^2} + \frac{1}{(x' - z)^2} \left[ 1 + \frac{x' - x}{R^2} \right] \right\} =$$

$$= \frac{\gamma(z, s) dz ds}{4\pi(x' - z)^2} \left( 1 + \frac{x' - x}{R^2} \right), \quad R^2 = (x' - x)^2 + (z' - z)^2$$

Let us replace the variable assuming that

then

$$x = x_0 + a(z) + \varepsilon a(z), \quad x' = x_0' + a' + \varepsilon' a'$$

$$\frac{x' - x}{a'} = p(z) + q(z)(1 + \varepsilon) + \varepsilon' - \varepsilon$$

$$p = \frac{x_0' - x_0}{a'}, \quad q = 1 - \frac{a}{a'}$$

where  $x_0$  is the coordinate of the leading edge in the  $z$  segment, while the dashes denote the values taken in the  $z'$  segment. It is evident that  $p$  characterizes the relative shift, while  $q$  characterizes the relative narrowing of the lines. The vortices of all lines, with the exception of the line with number  $k$  inside of which lies the point  $N'$ , will yield the velocity

$$dw_1 = \frac{d\varepsilon}{4\pi\lambda'} \left[ \int_{-1}^{z'-\varepsilon} \frac{\tilde{\gamma}(\xi, z)(1+f) dz}{(z' - z)^2} + \int_{z'+\varepsilon}^1 \frac{\tilde{\gamma}(\xi, z)(1+f) dz}{(z' - z)^2} \right], \quad (1)$$

where all  $z$  pertain to the  $L$  value and

$$f = \frac{p(z) + q(z)(1 + \varepsilon) + \varepsilon' - \varepsilon}{[(p + q(1 + \varepsilon) + \varepsilon' - \varepsilon)^2 + \lambda'^2(z' - z)^2]^{1/2}}, \quad (2)$$

$$\tilde{\gamma} = \gamma \frac{a}{a'}, \quad \lambda' = \frac{L}{a'}.$$

Vortices of the line with number  $k$  will elicit the velocity

$$-dw_2 = \frac{\gamma_k d\varepsilon}{2\pi(\xi' - \xi)} \frac{\lambda' \varepsilon}{\sqrt{(\xi' - \xi)^2 + \lambda'^2 \varepsilon^2}} + \frac{\gamma_k d\varepsilon}{2\pi\lambda' \varepsilon} \left( 1 + \frac{\xi' - \xi}{\sqrt{(\xi' - \xi)^2 + \lambda'^2 \varepsilon^2}} \right) =$$

$$= \frac{\gamma_k K_\varepsilon d\varepsilon}{2\pi(\xi' - \xi)} + \frac{\gamma_k d\varepsilon}{2\pi\lambda' \varepsilon} (1 + f'), \quad (3)$$

$$f' = (f)_{z=z'-\varepsilon}.$$

Let us use the approximation equality

$$\frac{2\sqrt{1-z'^2}}{\epsilon} - \left[ \int_{-1}^{z'-1} \frac{\sqrt{1-z^2} dz}{(z-z')^2} + \int_{z'+1}^1 \frac{\sqrt{1-z^2} dz}{(z-z')^2} \right] \approx \pi,$$

which will be more accurate the smaller the  $\epsilon$ . Then we will obtain

$$-dw_z = \frac{\gamma_k K_\epsilon d\xi}{2\pi(\xi' - \xi)} + \frac{\gamma_k d\xi}{4\lambda' \sqrt{1-z'^2}} (1+f') + \frac{d\xi}{4\pi\lambda'} \left[ \int_{-1}^{z'-1} \frac{\gamma^*(1+f') dz}{(z'-z)^2} + \int_{z'+1}^1 \right],$$

$$\gamma^* = \gamma_k(\xi) \frac{\sqrt{1-z^2}}{\sqrt{1-z'^2}}.$$

As a result, isolating the peculiarity and integrating, we will obtain

$$w = -\frac{1}{2\pi} \int_{-1}^{+1} \frac{\gamma_k d\xi}{\xi' - \xi} - \frac{1}{4\lambda' \sqrt{1-z'^2}} \int_{-1}^{+1} \gamma_k K(\xi, \xi', z') d\xi, \quad (4)$$

where

$$K = 1 + f' - \frac{2\lambda' (1 - K_\epsilon) \sqrt{1-z'^2}}{\pi(\xi' - \xi)} - \frac{1}{\pi} \left[ \int_{-1}^{z'-1} \frac{\sqrt{1-z^2} (f' + \tilde{f}') dz}{(z' - z)^2} + \int_{z'+1}^1 \right] -$$

$$- \frac{\sqrt{1-z'^2}}{\pi\gamma_k} \left[ \int_{-1}^{z'-1} \frac{(\tilde{\gamma} - \gamma^*) (1+f) dz}{(z' - z)^2} + \int_{z'+1}^1 \right]$$

is a regular function since  $1 - K_\epsilon$  disappears on the order of  $(\xi' - \xi)^2$  when  $\xi \rightarrow \xi'$ .

The integrals which are in the first brackets can be determined numerically with given  $\xi, \xi', z'$ . The integrals which are in the second brackets can be determined only after the values of  $\tilde{\gamma}$  in all sections can be determined in the first approximation.

On the basis of (4) the equation of impermeability at the point  $N'$  will assume the form of the following integral equation:

$$\frac{1}{2\pi} \int_{-1}^{+1} \frac{\gamma_k d\xi}{\xi' - \xi} = F(\xi, z') - \frac{1}{4\lambda' \sqrt{1-z'^2}} \int_{-1}^{+1} \gamma_k(\xi) K(\xi, \xi', z') d\xi, \quad (5)$$

where  $F$  is a given function. The equation of this form was obtained earlier, but now its kernel  $K$  has a simpler form, which is important for obtaining the approximate solution (see [1]). The integral equation (5) permits an innumerable number of solutions. From this number it is important to isolate a solution which corresponds to



the smooth flow near the trailing edge, which leads to the requirement that the density  $\gamma_k$  disappears on this edge. The disappearance should have a strictly defined order, since the principle value of the integral which enters the left side of equation (5) exists only in certain classes of functions, in which the integral permits the inversion. The inversion formulas have the following form (see [2] § 88)

$$\begin{aligned} \text{a)} \quad \gamma_k(l) &= \sqrt{\frac{1-l}{1+l}} \cdot \frac{2}{\pi} \int_{-1}^{+1} \sqrt{\frac{1+\xi'}{1-\xi'}} \cdot \frac{\Phi d\xi'}{\xi'-l}; \\ \text{b)} \quad \gamma_k(l) &= \sqrt{1-l^2} \cdot \frac{2}{\pi} \int_{-1}^{+1} \frac{\Phi d\xi'}{\sqrt{1-\xi'^2}(\xi'-l)}; \\ \text{c)} \quad \gamma_k(l) &= \frac{1}{\sqrt{1-l^2}} \cdot \frac{2}{\pi} \int_{-1}^{+1} \frac{\sqrt{1-\xi'^2} \Phi d\xi'}{\xi'-l}, \end{aligned}$$

where the right side of (5) is expressed in terms of  $\Phi$ .

After the inversion of the integral of equation (5) under the condition that the postulate of S. A. Chaplygin ( $\gamma(+1) \neq 0$ ) is satisfied according to formula a), we obtain

$$\gamma_k(l) = \bar{\gamma}_\infty - \frac{1}{2\pi k'} \sqrt{\frac{1-l}{1+l}} \int_{-1}^{+1} \gamma_k(\xi) \psi(\xi, l, z') d\xi, \quad (6)$$

where  $\bar{\gamma}_\infty$  corresponds to an infinite elongation and

$$\psi = \int_{-1}^{+1} \sqrt{\frac{1+\xi'}{1-\xi'}} \frac{K(\xi, \xi', z') d\xi'}{\xi'-l}.$$

One can attempt to solve this equation by the iterations method, having assumed that in the expression for  $K$  the  $\gamma = \gamma^*$  in the first approximation. Then equation (6) will transform into the Fredholm equation of the second order with the regular kernel. In order to solve it by the iterations method, it is convenient to present the solution in the following form

$$\gamma_k(\xi) = \bar{\gamma}_k(\xi) + \Delta\gamma_k = \gamma_k(l) \sqrt{\frac{1+l}{1-l}} \sqrt{\frac{1-\xi}{1+\xi}} + \Delta\gamma_k,$$

where  $\Delta\gamma_k(\pm 1) = 0$ .

After the substitution, the equation (6) will assume the form



$$\gamma_k(l) = \frac{\gamma_{-k} - \Delta\gamma_{-k}}{1 + \frac{1}{2\pi\lambda'} \cdot \frac{1}{\sqrt{1-z'^2}} J_1(l)}, \quad (7)$$

where

$$\Delta\gamma_{-k} = \frac{1}{2\pi\lambda' \sqrt{1-z'^2}} \sqrt{\frac{1-l}{1+l}} \int_{-1}^{+1} (\gamma_k - \bar{\gamma}_k) \phi d\xi,$$

$$J_1(l, z') = \int_{-1}^{+1} \sqrt{\frac{1-l}{1+l}} \phi d\xi.$$

When solving equation (7) by the iterations method one should assume that  $\gamma_k$  is equal to  $\bar{\gamma}_k$  ( $\Delta\gamma_{-k}=0$ ) in the zero approximation. This enables one to determine the  $\gamma_k$  in all sections  $z'_k$  in the first approximation, i. e., to determine  $\gamma(\xi, z)$ , then find the differences  $\bar{\gamma}(\xi, z) - \gamma^*(\xi, z)$  and  $\Delta\gamma_k$  and proceed with the calculation of the second approximation. The solution becomes completely elementary if one uses the Vayzinger [Weisinger] hypothesis and one assumes that  $\xi' - \xi$  is equal to unity when calculating  $J_1$  and  $\Delta\gamma$  and assume that  $\xi'$  is equal  $-0.5$ . From formula (7) it is evident that  $\gamma(l, z')$  vanishes just like  $(1 - z'^2)^{0.5}$  when  $z' \rightarrow \pm 1$ , and the derivative  $\frac{\partial \gamma}{\partial \xi}$  goes to infinity just as  $(1 - z'^2)^{-0.5}$  when  $z' \rightarrow \pm 1$ . We can prove to ourselves that these boundary conditions are a direct consequence of the vortex method. Actually the vortex sheet, which corresponds to the curve  $\xi = \text{const}$ , by intersecting with the Trefftz [sp. unconfirmed] plane at infinity which is perpendicular to it yields a rectilinear segment with the span  $(-1, 1)$  which is covered by a vortex layer with density  $\frac{\partial \gamma}{\partial \xi}$ . The inductive velocity at the points of this segment is equal to

$$w(z') = \frac{1}{2\pi} \int_{-1}^{+1} \frac{\partial \gamma}{\partial \xi} \cdot \frac{dz}{z' - z}.$$

Since due to symmetry the circulation near the examined segment is zero, the interval should be inverted according to formula c) which in this case will have the form

$$\frac{\partial \gamma}{\partial s} = \frac{2}{\pi} \frac{1}{\sqrt{1-s^2}} \int_{-1}^{+1} \frac{\sqrt{1-s'^2} w(s') ds'}{s'-s},$$

which corresponds to the boundary conditions formulated above. From the aforesaid it is evident that the boundary conditions for  $\gamma$ , both at the leading edge and the trailing edge and also on lateral edges, are the consequence of the introduction of the vortex system for which the appearance of singular intervals which permit strictly defined conditions at the boundaries of the integration region is characteristic.

§ 2. Substituting the integral equation with a system of algebraic equations. This operation should be carried out, understandably, while adhering to the boundary conditions indicated above. The presence of the singular integral in equation (5) forces one to seek the solution in the form

$$\gamma_k = A_0 \sqrt{\frac{1-\xi}{1+\xi}} + \eta_k, \quad (8)$$

where  $\eta_k(\pm 1)$  should vanish just as  $\sqrt{1-\xi^2}$  when  $\xi \rightarrow \pm 1$ . Substituting (8) in (5) and eliminating the subscript  $k$ , we obtain

$$\frac{A_0}{2} \left[ 1 + \int_{-1}^{+1} \sqrt{\frac{1-\xi}{1+\xi}} K_1 d\xi \right] + \frac{1}{2\pi} \int_{-1}^{+1} \frac{\eta d\xi}{\xi' - \xi} = F(\xi', z') - \frac{1}{2} \int_{-1}^{+1} \eta K_1 d\xi, \quad (9)$$

where  $K_1 = K(\xi, \xi', z') : 2\lambda'(1-z'^2)^{0.5}$ .

To calculate the last integrals in the left and right sides of equation (9) we divide the interval of integration into  $N$  partial intervals  $(-1, \xi_1), (\xi_1, \xi_2) \dots (\xi_{N-1}, 1)$ . We will assume that the function  $K(\xi, \xi', z')$  is known in each partial interval, having assumed that in the first approximation  $\tilde{\gamma} \approx \gamma^*$ . In the first and last intervals we approximate  $\eta$  according to the parabolic law:

$$\eta_1 = \bar{\eta}_1 \sqrt{\frac{1-|\xi|}{1+|\xi_1|}}, \quad \eta_N = \bar{\eta}_{N-1} \sqrt{\frac{1-\xi}{1-\xi_{N-1}}},$$

and in the remaining - according to the linear law:

$$\eta_m = \bar{\eta}_{m-1} + (\bar{\eta}_m - \bar{\eta}_{m-1}) \frac{\xi - \xi_{m-1}}{\xi_m - \xi_{m-1}},$$

where  $\bar{\eta}$  - values of  $\eta$  at the points of division. In this approximation the second interval on the left side of equation (9) assumes the form

$$\frac{1}{2\pi} \int_{-1}^{+1} \frac{\eta d\xi}{\xi' - \xi} = \frac{1}{2\pi} \sum_{m=1}^{N-1} \int_{\xi_{m-1}}^{\xi_m} \frac{\eta_m d\xi}{\xi' - \xi} = \sum_{m=1}^{N-1} \bar{\eta}_m C'_m(\xi'),$$

where  $C'_m$  represented by the integrals are calculated very simply, analytically for any  $\xi'$ , including the interval containing the singularity. The integral on the right side is brought to the following form exactly in the same way

$$\int_{-1}^{+1} \eta K_1 d\xi = \sum_{m=1}^{N-1} \int_{\xi_{m-1}}^{\xi_m} \eta_m K_1 d\xi = \sum_{m=1}^{N-1} \bar{\eta}_m C''_m(\xi').$$

The integrals which enter the  $C''_m$  can be calculated using the formulas of mechanical quadratures, since they do not have peculiarities. As a result, equation (9) will proceed to the following system of algebraic equations:

$$A_0 C_0(\xi', z') + \sum_{m=1}^{N-1} \bar{\eta}_m C_m(\xi', z') = F(\xi', z'). \quad (10)$$

In this case the  $A_0$ ,  $\bar{\eta}_1$ ,  $\bar{\eta}_2$ , ...,  $\bar{\eta}_{N-1}$  are unknown. Their number is equal to the number of intervals and, therefore, if inside each of these we take a point with the coordinate  $\xi'$ , we will have a system which will enable us to determine all the unknowns. The density  $\mathcal{P}$  is determined by formula (8). This approximate solution will satisfy the required boundary conditions of the vortex method.

§ 3. Method of discrete vortices. There are two concepts regarding this method. One of these is the Folkner concept and it entails the fact that the supporting surface is broken down into a series of lines which are parallel to the direction of motion. Each of the lines is covered by a finite number of  $\Gamma$ -like vortices with the circulations  $\Gamma_{mn}$ , where  $m$  - line number,  $n$  - point number in this



line. In each section the values of  $\Gamma_{mn}$  join the coefficients of expansion of the density  $\gamma_m$  in the series

$$\gamma_m = \sqrt{1-z_m^2} \left[ A_0(z_m) \operatorname{ctg} \frac{\theta}{2} + \sum_{n=1}^{\infty} A_n \sin n\theta \right], \quad (11)$$

where

$$\xi = -\cos \theta, A_n = A_{n0} + A_{n1}z_m + A_{n2}z_m^2 + \dots, n=0, 1, 2, \dots$$

The connection between the  $\Gamma_{mn}$  and  $A_{nn}$  is established by equating the total circulations originating from the discrete and continuous distributions in each section. The same thing is done with the inductive velocities at the assigned points. In this way all  $\Gamma_{mn}$  can be expressed in terms of  $A_{nn}$  and, consequently, total inductive velocity. Folkner does not write the integral equation explicitly, and for determining the  $A_{nn}$  he uses the system of equations which expresses the condition of impermeability at the various points of the sections used. The boundary conditions which correspond to the vortex method are satisfied in this method.

Another approach towards the method of discrete vortices consists of the fact that the expansion (11) is not introduced and the circulations of  $\Pi$ -shaped vortices (the usual rectangular or skewed) remains under the conditions of impermeability so that what we get is a system of algebraic equations not relative to  $A_{nn}$ , as Folkner has, but relative to  $\Gamma_i$ . This system has the form

$$\sum_{i=1}^m \Gamma_i K_{ij} = F_j, \quad (12)$$

where  $i$  - vortex number,  $j$  - number of a point, in this case the numeration proceeds successively. Furthermore, segments of lifting vortices are located at  $0.25h$ , and the condition of impermeability is fulfilled at points  $0.75h$ , where  $h$  - length of partial intervals into which the cross-sections  $z=\text{const}$  are broken down.

Using the example of a two-dimensional stationary flow which will be described by equation (5) with the zero value of the integral on the right side of ( $\lambda=\infty$ ), we can see that system (12) cannot give us a correct solution for the problem. Actually, this system obtained from an integral equation will have the form



$$\sum_{i=1}^n K_{ij} \int_{\xi_{i-1}}^{\xi_i} \gamma d\xi = F_j, \quad K_{ij} = (\xi_j' - \xi_i')^{-1}. \quad (13)$$

Firstly, the substitution of the kernel using formula (13) in the interval where it becomes unlimited ( $\xi_i'$  - coordinates of discrete vortices) is totally inadmissible and, secondly, even we accept structure (12) for the initial system the theory of integral equations says the following concerning the correspondance between (12) and (13) (see [3], pg. 12): "even if the determinant of system (12) does not equal zero and the system has one solution, system (13) will have an infinitely large number of solutions, since only the average values of the  $\gamma$  function in the intervals  $(-1, \xi_1)$ ,  $(\xi_1, \xi_2)$ , etc. are determined uniquely".

It is known that in order to select a particular solution from an infinite number of solutions which would correspond to the physical conditions, it is necessary to put forth the boundary conditions which would correspond to the vortex method. Their satisfaction must be guaranteed, but system (12) which does not reflect to any degree the conditions at the boundaries does not fulfil this. It is necessary to remember that the approximating function must satisfy the same boundary conditions as the approximation function. It is assumed that the vanishing of the vortex density at the trailing edge in the limit will be fulfilled automatically if the last control points in each section are taken between the trailing edge and the last discrete vortex. There are no substantiations for this assertion. In this case it is necessary to remember that what is important here is not just the vanishing, but the vanishing of the rigidly defined order. The selection of the last point does not correspond to the structure of a particular integral and, therefore, cannot supply the necessary order of disappearance. Actually, this integral exists only in the concept of the principal value which is obtained for the point lying between the two segments of the vortex layer to which corresponds the point lying between two vortices in the discrete scheme. The "last" point does not satisfy this condition. Thus, system (12) cannot replace the initial integral equation with the boundary conditions

which correspond to it. Therefore, the assertion that the method of discrete vortices in the theory of the lifting surface " leads, in all cases, to the substitution of two- or three-dimensional integrodifferential equations with the systems of algebraic equations" (see [4], pg. 222) is not true. Such a substitution for the equations of the first order without the guaranteed satisfaction of the rigidly defined boundary conditions is not lawful. The solution depends on the nature of distribution of the vortices at the calculated points which is assigned quite arbitrarily.

#### Bibliography

Submitted  
11 Jan 1973

1. Поляхов Н. Н. и Пастухов А. И. Вихревая теория несущей поверхности.— Вестн. Ленингр. ун-та, 1968, № 19, с. 121—131.
2. Мусхелишвили Н. И. Сингулярные интегральные уравнения. М.—Л., Гостехиздат, 1946. 448 с.
3. Трикоми. Интегральные уравнения. М., ИЛ, 1960. 200 с.
4. Белоцерковский С. М., Скрипач Б. К., Табачников В. Г. Крыло в нестационарном потоке газа. М., «Наука», 1971. 767 с.

#### Summary

The paper represents the simplification of the article [1]. The integral equation according to lifting surface theory has a form (5) which can be transformed to the form (7). It is possible to solve (7) by iteration. The second paragraph gives a system of linear equations (10) which is approximately equivalent to integral equation (5). The vorticity distribution per unit chord  $\gamma$  is given by (8),  $\eta(\pm 1, z) = 0$ .

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